4-2 Videos Guide

4-2a

Definitions: (definite integral and integrable)

- The definite integral of f from a to b is
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$
where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$, provided the limit exists. If the limit does exist, we say that f is integrable on $[a, b]$.

Exercise:

- Evaluate the definite integral.
$$\int_1^4 (x^2 - 4x + 2)\,dx$$

- Summation formulas
  - $\sum_{i=1}^{n} c = cn$
  - $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
  - $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
  - $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

4-2b

Exercise:

- Express the limit as a definite integral on the given interval.
$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i\sqrt{1 + x_i^3}\,\Delta x; \quad [2, 5]$$

- Properties of the definite integral
  - $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
  - $\int_a^a f(x)\,dx = 0$
  - $\int_a^b c\,dx = c(b-a)$, where c is any constant
  - $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
  - $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$
  - $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$

Exercise:

Evaluate the integral by interpreting it in terms of areas.
$$\int_{-5}^{5} \left(x - \sqrt{25 - x^2}\right) dx$$

4-2c

- Comparison properties of the definite integral
    - If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq 0$ (which gives the area under the graph of f from a to b)
    - If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$
    - If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

Exercises:

- Use a comparison property of the definite integral to estimate the value of the integral.
  $$\int_0^3 \frac{1}{x+4}\,dx$$